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EFFECT OF DIAMETER ON CRITICAL WEISSENBERG NUMBERS FOR POLYACRYLAMIDE SOLUTIONS IN TURBULENT PIPE FLOW

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Abstract—The influence of pipe diameter on the critical Weissenberg numbers for friction and heat transfer for aqueous polyacrylamide solutions in turbulent pipe flow was investigated in three different size tubes: 0.98, 1.30 and 2.25 cm I.D. New measurements taken with a 1000 wppm aqueous polyacrylamide solution in 0.98 and 1.30 cm I.D. tubes were compared with earlier measurements carried out using a 1500 wppm solution in a 2.25 cm I.D. tube. A comparison of these results indicate that the effect of diameter on the critical Weissenberg numbers for friction and heat transfer is not important. The current experimental results confirmed the earlier finding that the critical Weissenburg numbers based on the Powell–Eyring model are of the order of 5–10 for friction and 200–250 for heat transfer.

NOMENCLATURE

c_p, specific heat of fluid;
d, tube diameter;
f, Fanning friction factor, $f = \tau_w/(\rho V^2/2)$;
h, convective heat transfer coefficient;
j_H, dimensionless heat transfer factor, $St\, Pr_a^{2/3}$;
k_t, thermal conductivity of fluid;
l, total tube length;
Nu, Nusselt number, hd/k_t;
Pr_a, Prandtl number based on the viscosity at wall, $\eta c_p/k_t$;
Re_a, Reynolds number based on the viscosity at wall, $\rho V d/\eta$;
St, Stanton number, $Nu/(Re_a Pr_a)$;
V, average velocity;
Ws, Weissenberg number, $\lambda V/d$;
Ws_{cf}, critical Weissenberg number for friction;
Ws_{ch}, critical Weissenberg number for heat transfer;
x, axial distance.

Greek symbols

$\dot{\gamma}$, shear rate;
η, apparent viscosity evaluated at wall;
η_0, zero shear rate viscosity;
η_∞, apparent viscosity at infinite shear rate;
ρ, density of fluid;
τ_w, wall shear stress;
λ, characteristic time of fluid.

INTRODUCTION

EVER since Virk *et al.* [1] reported the existence of the maximum drag reduction asymptote for drag reducing viscoelastic fluids, there have been attempts to identify the corresponding minimum heat transfer asymptote. Although a number of investigators [2–9] have reported what they considered to be the minimum heat transfer asymptote the reported values varied widely from investigation to investigation. These variations may be the results of such factors as differences in the thermal entrance lengths [8, 10, 11], polymer concentration [3, 5, 7, 11], the chemistry of the solvent [12–14] and the level of mechanical degradation [15–17].

Yoo and Hartnett [18] reported that purely viscous non-Newtonian fluids require approximately the same thermal entrance length as Newtonian fluids, while the thermal entrance length for viscoelastic fluids exceeded the 110 pipe diameters test section used in their study. This interesting result was in agreement with the analytical predictions of Dimant and Poreh [19]. Subsequent experimental studies revealed that the thermal entrance length for viscoelastic fluids may be as long as 400–500 diameters for concentrated polymer solutions [11]. Since few, if any, of the investigations used test sections of such lengths, it is understandable that results varied from one investigator to another.

Polymer concentration is another factor since, in general, the friction factors and the heat transfer coefficients decrease (to a certain point) with increasing polymer concentration. Another complicating feature of these viscoelastic fluids is degradation which occurs as a result of destruction of the polymer bonds caused by the shearing stresses encountered during the flow process. Such degradation is associated with an increase in friction and heat transfer. The effects of the solvent chemistry on the friction and heat transfer are even more complicated than the effects of either polymer concentration or polymer degradation.

Experimental studies have demonstrated that the effects of polymer concentration [20, 21], solvent chemistry [22, 23] and polymer degradation [16] are related to the elasticity of the viscoelastic fluids. These effects may be represented by a dimensionless parameter such as the Weissenberg number that characterizes the ratio of the elastic force to the viscous force. The Weissenberg number is defined as

$$Ws = \lambda V/d \quad (1)$$

where λ is a characteristic time of the fluid and is a measure of the elasticity of the fluid; V/d is a characteristic shear rate.

Generally the friction factor decreases with increasing concentration up to a certain value beyond which further increases in the concentration have no influence and the friction factor has reached its minimum asymptotic value. The same behavior is found for heat transfer, except that much higher polymer concentrations are required to reach the asymptotic heat transfer behavior. Since the polymer concentration is related to the elasticity of the fluid this behavior suggests that there may be critical Weissenberg numbers for friction and heat transfer. If the Weissenberg number is greater than the critical Weissenberg number of friction and heat transfer, the friction factors and dimensionless heat transfer coefficients reach minimum asymptotes and each can then be expressed as a function of Reynolds number only for fully established hydrodynamic and thermal conditions. The observation that it requires a higher polymer concentration to attain the minimum heat transfer asymptote than the necessary concentration to give the minimum drag asymptote implies that more elastic force is needed to reach the minimum heat transfer asymptote.

In the case where a concentrated aqueous solution of a high molecular weight polymer is circulated continuously in a flow loop, degradation occurs and the heat transfer coefficient departs from its asymptotic values after a certain time; subsequently the friction factor increases after an additional period of circulation [17, 24]. Therefore, it is expected that the critical Weissenberg number of heat transfer may be larger than that of friction.

Kwack *et al.* [24] conducted turbulent heat transfer and pressure drop measurements with aqueous polyacrylamide solutions in two different modes: the once-through system and recirculating flow system. The critical Weissenberg numbers for friction and for heat transfer were found to be 5–10 and 200–250 respectively for the Reynolds number range from 20000 to 30000. In that study only one circular tube with an inside diameter of 1.30 cm. I.D. was used.

It can be readily shown that the Weissenberg number is inversely proportional to the square of the diameter of the circular tube for a fixed value of the Reynolds number. Consequently a sensitive test of the earlier reported critical Weissenberg numbers can be obtained by varying the pipe diameter. It is the purpose of this paper to carry out such an investigation.

Since the elastic behavior of viscoelastic fluids can be changed by mechanical shearing resulted from continuous recirculation in a closed flow loop, Weissenberg number effects can be examined in degradation runs. To investigate the diameter effects mentioned above, simultaneous measurements of the friction factor and heat transfer coefficient for drag reducing polyacrylamide solutions were carried out in a recirculating flow system using three circular tubes of different diameters. For each tube, the measurement of the fluid viscosity–shear rate behavior as a function of circulation time allows the calculation of a characteristic time using the Powell–Eyring model [25]. In this way the Weissenberg number may be determined as a function of time.

EXPERIMENT

Three circular tubes having inside diameters of 0.98 cm (l/d = 620), 1.30 cm (l/d = 475) and 2.25 cm (l/d = 285) were used in the current study. A schematic diagram of the recirculating flow system is shown in Fig. 1. A positive displacement Moyno pump discharged the test fluid into one of the three horizontal tubes as selected. In the present flow system, the

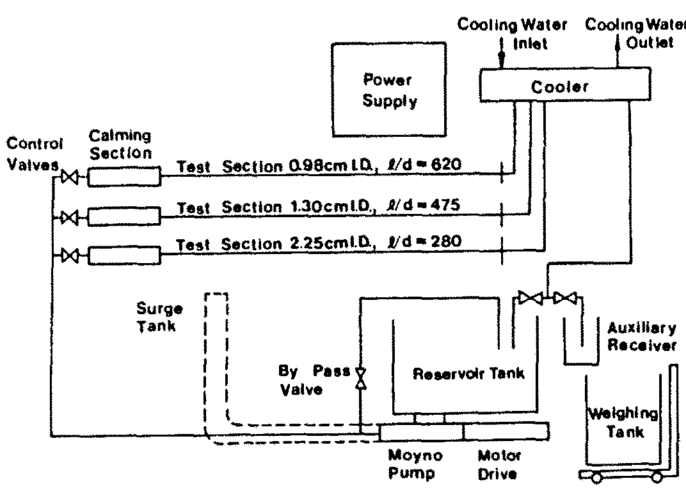

FIG. 1. Schematic diagram of the recirculating flow system.

hydrodynamic and thermal entrance regions develop simultaneously from the beginning of the test section. To provide the boundary condition of constant heat flux at the wall, the test tubes were heated electrically by a DC power supplier. Some thirteen pressure taps were installed along the length of each tube. Forty 30-gage copper–constantan thermocouples were cemented with copper oxide along the length of each tube to measure the local outside wall temperature. A shell and tube type heat exchanger was connected at the end of the flow loop to maintain constant inlet temperature of test fluid.

The pressure drops were measured by a set of parallel water manometers of length 1.8 m with an adjustable air pressure head to offset the pressure in the tube. Thermocouple readings and the voltage drops across the heat transfer section of each tube were measured with a precision digital voltmeter having 1 μV resolution. Flow rates were measured by direct weighing.

The fluid was continuously circulated in the flow loop for a period of about 70 h for each degradation run. The measurements of heat transfer coefficient and friction factor were conducted simultaneously at regular time intervals while at the same time fluid samples were taken from the flow loop for viscosity measurements.

Calibration measurements of pressure drop and heat transfer for the flow loop were made with tap water.

PROPERTIES OF POLYMER SOLUTIONS

The polymer solutions used in the current study were 1000 wppm and 1500 wppm aqueous solutions of polyacrylamide (Separan AP-273) from the Dow Chemical Company dissolved in Chicago tap water [14]. The 1000 wppm Separan solutions were used for degradation runs in circular tubes of 0.98 and 1.30 cm I.D. while the 1500 wppm Separan solution results were obtained in the 2.25 cm I.D. tube [17].

The apparent viscosities of samples of test fluid removed from the recirculating flow system at regular time intervals were measured using the Weissenberg rheogoneometer (R-18) with Couette geometry as well as using the capillary tube viscometer (0.05334 cm I.D. and l/d = 375).

Figures 2 and 3 show the apparent viscosity of the 1000 wppm Separan solutions taken at different hours of shear during the degradation run in the recirculating flow system. Figure 4 shows the apparent viscosity of the 1500 wppm Separan solution as a function of shear rate. The zero shear rate viscosity is seen to continuously decrease with shearing time, which reflects the rupturing of polymer bonds [26] resulting in a continuous loss of elasticity of the fluid. The rate of decrease appears to be highest during the initial period. It is interesting to note that the viscosity at high shear rates (10^3–10^4) remains relatively constant during the entire test program. Such high shear rates are in the range of the wall shear rates encountered in the flow loop.

Figure 5 shows the values of the zero shear rate viscosity of each solution as a function of hours of shear. The zero shear rate viscosity of the 1000 wppm Separan solution for the 0.98 cm I.D. tube at zero hours of shear is almost half of that for the 1.30 cm I.D. tube for the same concentration. This underscores the necessity of directly measuring the viscosity–shear rate behavior for aqueous polymer solutions since minor variations in the chemistry of the solvent and of the solute may cause substantial changes in the rheology.

The Powell–Eyring model was used to calculate the characteristic times for all test fluids using the values of

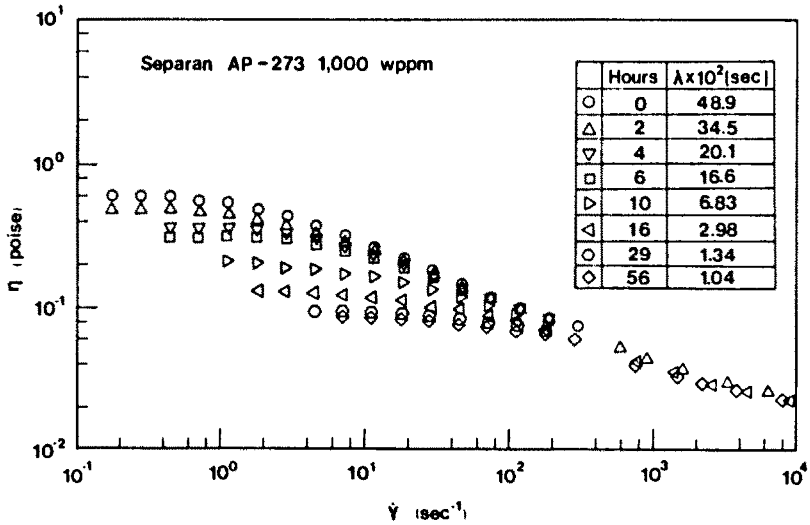

FIG. 2. Apparent viscosity vs shear rate for 1000 wppm Separan solution in 0.98 cm I.D. tube.

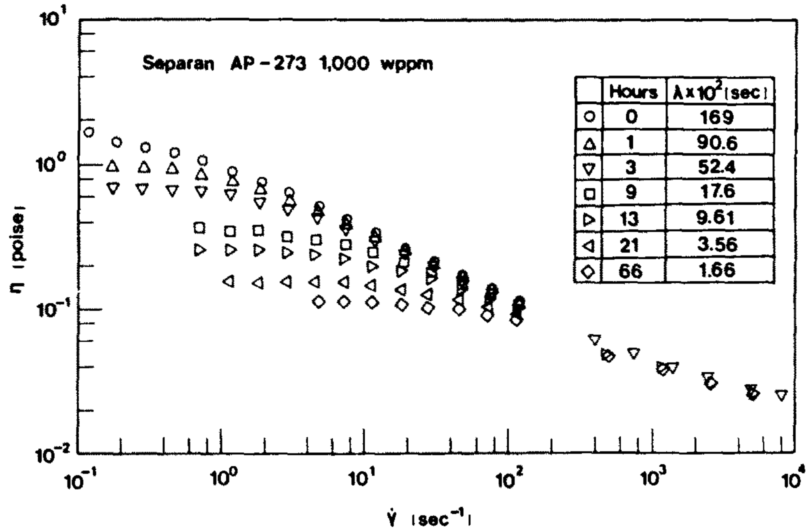

FIG. 3. Apparent viscosity vs shear rate for 1000 wppm Separan solution in 1.30 cm I.D. tube.

steady shear viscosity only. The Powell–Eyring model is defined as follows [25]:

$$\eta = \eta_\infty + (\eta_0 - \eta_\infty)\left[\frac{\sinh^{-1}\lambda\dot{\gamma}}{\lambda\dot{\gamma}}\right] \qquad (2)$$

where λ is the characteristic time. Since the zero shear rate viscosity, η_0, and viscosity at infinite shear rate, η_∞, are measured, the Powell–Eyring model offers a simple procedure for calculating the characteristic time. This model may be applicable to dilute polymer solutions because it includes the viscosity at infinite shear rate, η_∞. Figure 6 shows the values of characteristic time of each solution as a function of hours of shear.

RESULTS AND DISCUSSION

The degradation runs for the pressure drop and heat transfer measurements with 1000 wppm and 1500 wppm Separan solutions were conducted in the recirculating flow system. The friction factor and heat transfer coefficient were measured simultaneously at regular time intervals. The pressure drop measurements were made at x/d values larger than 110 to obtain the hydrodynamically established Fanning friction factor [27]. The heat transfer coefficients measured at x/d ratio equal to 430 are presented in Figs. 7 and 8 together with the Fanning friction factors. Since the l/d ratio of the 2.25 cm I.D. tube is only 285, the dimensionless heat transfer factor j_H is much higher

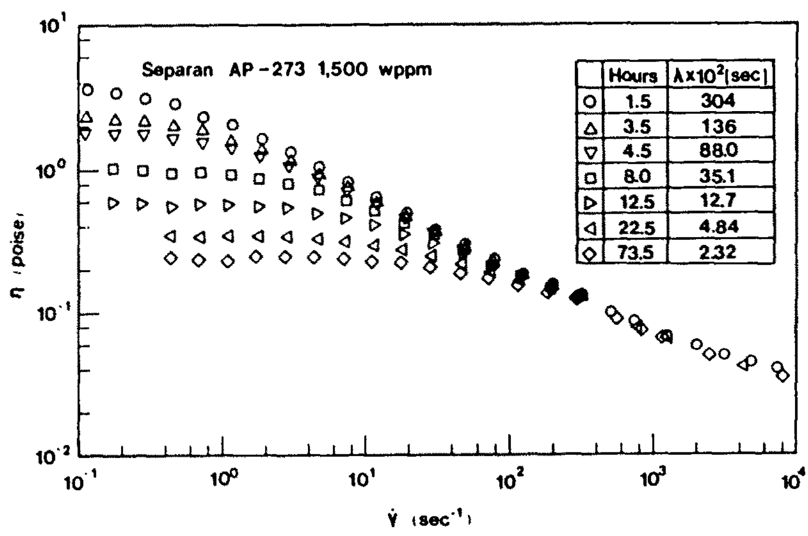

FIG. 4. Apparent viscosity vs shear rate for 1500 wppm Separan solution in 2.25 cm I.D. tube [17].

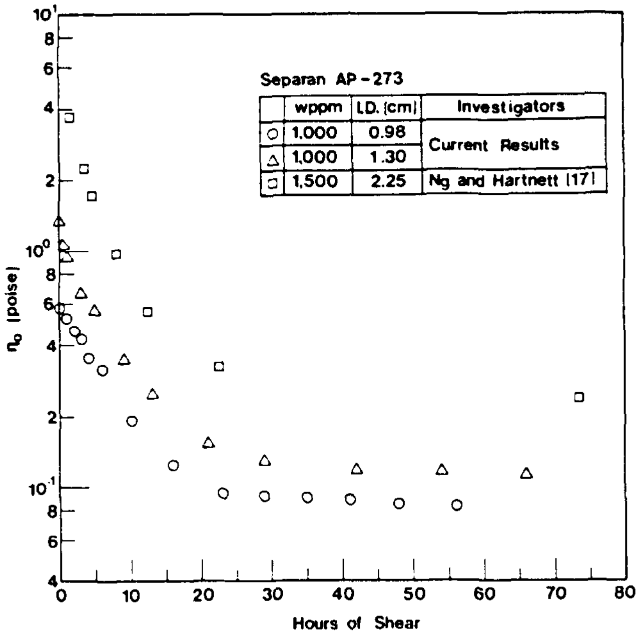

FIG. 5. Zero shear rate viscosity vs hours of shear for 1000 wppm and 1500 wppm Separan solutions.

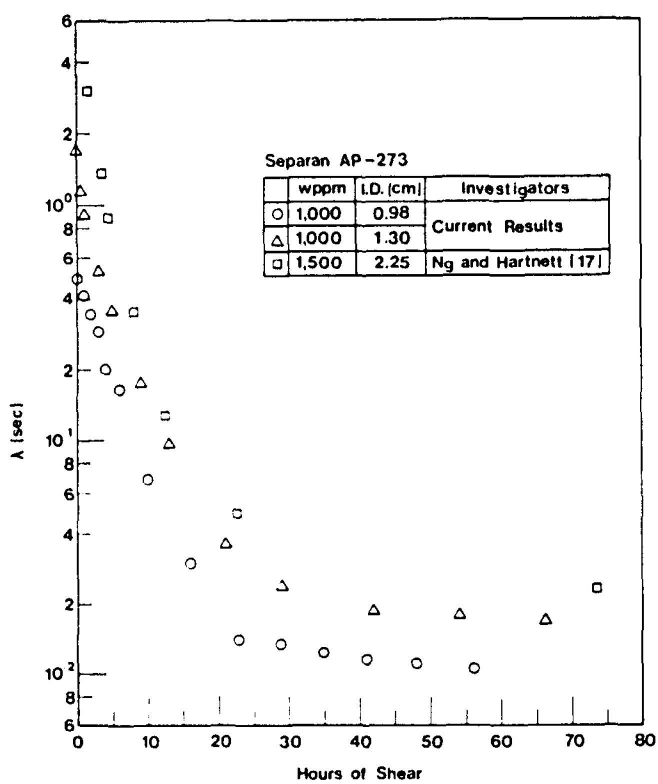

FIG. 6. Characteristic time vs hours of shear for 1000 wppm and 1500 wppm Separan solutions.

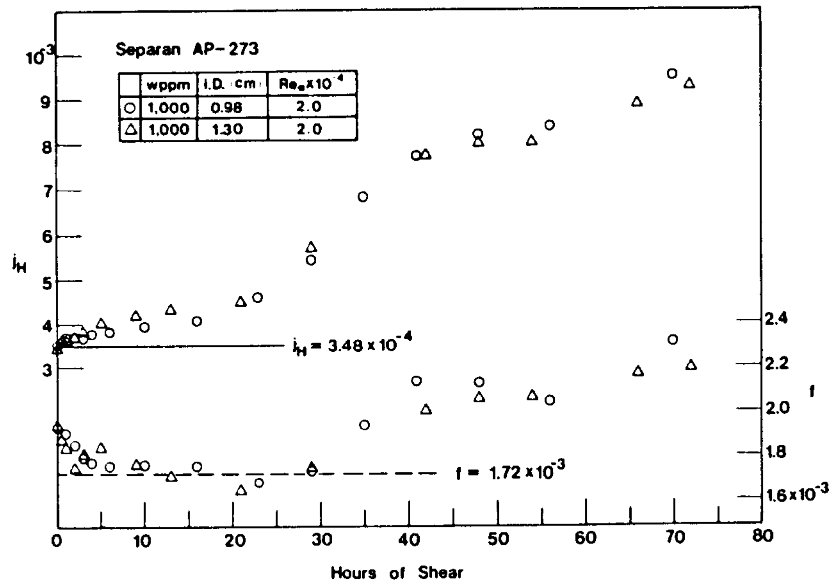

FIG. 7. Fanning friction and dimensionless heat transfer factors vs hours of shear for Re_a of 20000.

than minimum heat transfer asymptote at the corresponding Reynolds number; therefore these results were not included. Using the previously reported empirical relations, i.e. $f = 0.20\,Re_a^{-0.48}$ [28] and $j_H = 0.03\,Re_a^{-0.45}$ [11, 29] minimum drag asymptotic values and the minimum heat transfer asymptotic values corresponding to each Reynolds number are included in Figs. 7 and 8 for comparison.

The most striking feature of these figures is that the dimensionless heat transfer factor j_H departs from its asymptotic values much earlier than the friction factor does. The results of the degradation run in the 0.98 cm I.D. circular tube is almost the same as that of the degradation run in the 1.30 cm I.D. tube. At a Reynolds number of 20000, it takes about 30 h of shear before the friction factor begins to increase above its asymptotic value, while it takes only 3 h of shear before the dimensionless heat transfer factor j_H departs from its minimum value. For a Reynolds number of 30000, it takes almost the same hours of shear before the friction factor begins to increase above its asymptotic value, but it takes 5 h of shear before the dimensionless heat transfer factor j_H does. Figure 8 also includes the pressure drop measurements of the degradation run in

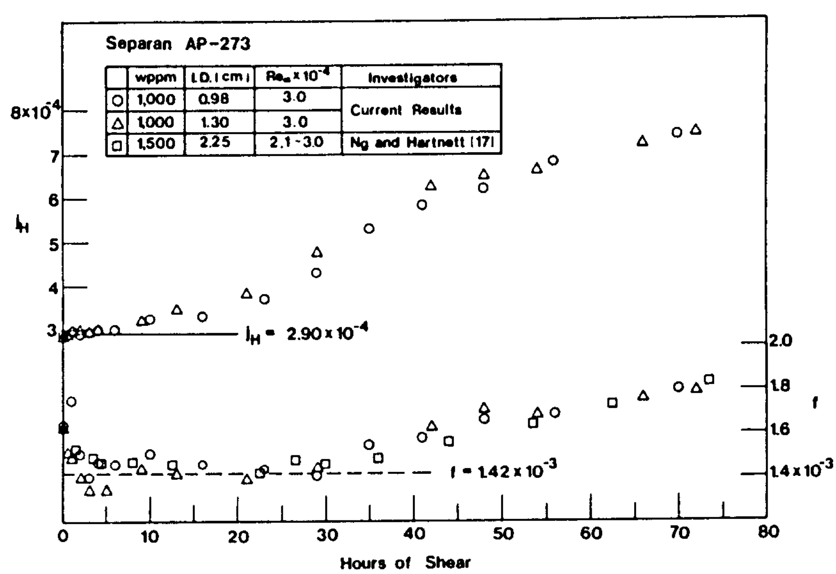

FIG. 8. Fanning friction and dimensionless heat transfer factors vs hours of shear for Re_a of 30000.

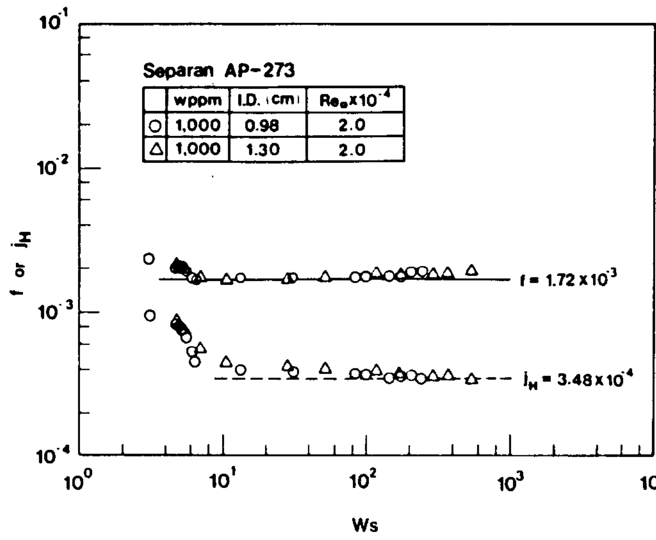

FIG. 9. Fanning friction and dimensionless heat transfer factors vs Ws for Re_a of 20000.

the 2.25 cm I.D. circular tube reported earlier [17]. The friction factor for the 2.25 cm I.D. tube starts to increase at approximately 40 h of shear.

These pressure drop and heat transfer measurements are replotted in Figs. 9 and 10 as functions of the Weissenberg number. These two figures support the earlier finding that the critical Weissenberg number for heat transfer is larger than the critical Weissenberg number for friction [24]. Furthermore, the results of the friction and dimensionless heat transfer factors vs the Weissenberg numbers are independent of the pipe diameter. In particular, the results of the friction factor in Fig. 10 which includes 3 different size tubes shows clearly that the diameter does not affect the critical Weissenberg number for friction. From these results it is concluded that the influence of pipe diameter on the friction factor and the dimensionless heat transfer factor j_H is adequately accounted for in the Weissenberg numbers and the Reynolds numbers for fully established conditions. The critical Weissenberg number for the Fanning friction factor is of the order of 5-10 for fully established turbulent flow of aqueous polyacrylamide solutions in a circular tube over the Reynolds number range of 20000 to 30000. The corresponding critical Weissenberg number for the dimensionless heat transfer factor j_H for fully established thermal conditions is of the order of 200-250. A weak dependency on Reynolds number is apparent, with the critical Weissenberg numbers increasing slightly with Reynolds number.

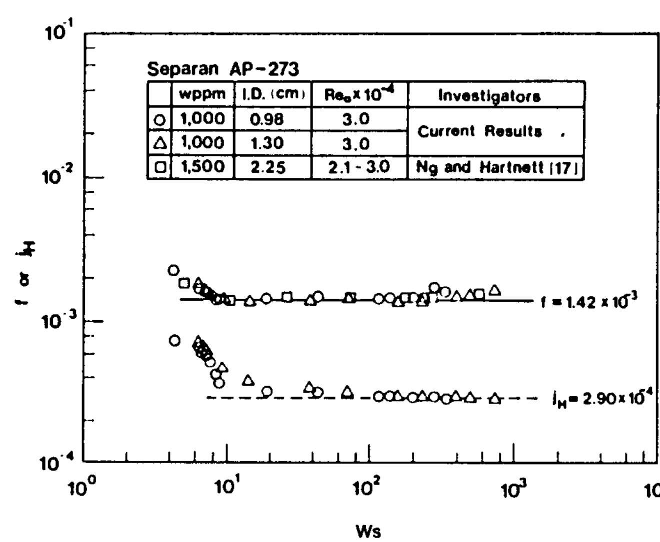

FIG. 10. Fanning friction and dimensionless heat transfer factors vs Ws for Re_a of 30000.

CONCLUSIONS

(1) The current experimental results provided additional confirmation of the existence of critical Weissenberg numbers for friction and heat transfer, above which the fully established friction factor and heat transfer coefficient reach minimum asymptotic values.

(2) The diameter does not affect the critical Weissenberg numbers for friction and heat transfer.

(3) The critical Weissenberg numbers for friction and heat transfer for dilute aqueous polyacrylamide solutions are as follows:

$$Ws_{cf} = 5\text{–}10 \quad \text{for } 20000 < Re_a < 30000,$$

$$Ws_{ch} = 200\text{–}250 \text{ for } 20000 < Re_a < 30000.$$

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EFFET DU DIAMETRE SUR LES NOMBRES CRITIQUES DE WEISSENBERG POUR LES SOLUTIONS DE POLYACRYLAMIDE EN ECOULEMENT TURBULENT DANS UN TUBE

Resume—L'influence du diamétre du tube sur les nombres critiques de Weissenberg pour le frottement et le transfert thermique dans des solutions aqueuses polyacrylamide en écoulement turbulent est étudiée pour trois diamètres intérieurs de tube: 0,98 cm, 1,30 cm et 2,25 cm. De nouvelles mesures pour une solution à 1000 ppm en masse dans les tubes de 0,98 cm et 1,30 cm sont comparées avec des mesures antérieures effectuées sur une solution à 1500 ppm dans un tube de 2,25 cm. Une comparaison montre que l'effet du diamètre sur les nombres critiques de Weissenberg pour le frottement et le transfert thermique n'est pas important. Les résultats expérimentaux confirment que les valeurs précédemment données des nombres critiques de Weissenberg basées sur le modèle Powell-Eyring sont de l'ordre de 5–10 pour le frottement et 200–250 pour le transfert thermique.

DER EINFLUSS DES DURCHMESSERS AUF KRITISCHE WEISSENBERG-ZAHLEN FÜR POLYACRYLAMID-LÖSUNGEN BEI TURBULENTER ROHRSTRÖMUNG

Zusammenfassung—Der Einfluß des Rohrdurchmessers auf die kritischen Weissenberg-Zahlen für Reibung und Wärmeübergang wurde für wäßrige Polyacrylamid-Lösungen bei turbulenter Rohrströmung in drei Rohren mit 0,98; 1,30 und 2,25 cm Innendurchmesser untersucht. Neue Messungen mit einer 1000 ppm wäßrigen Polyacrylamid-Lösung in Rohren mit 0,98 und 1,30 cm Innendurchmesser wurden mit früheren Messungen mit einer 1500 ppm Lösung in einem Rohr mit 2,25 cm Innendurchmesser verglichen. Der Vergleich zeigt, daß der Einfluß des Durchmessers auf die kritischen Weissenberg-Zahlen für Reibung und Wärmeübergang unbedeutend ist. Die neuen experimentellen Ergebnisse bestätigten frühere Erkenntnisse, daß die kritischen Weissenberg-Zahlen nach dem Powell-Eyring-Modell für Reibung im Bereich 5–10 und für Wärmeübergang im Bereich 200–250 liegen.